TITLE: [Insert Course Title.]
[Tip: The course name should reflect the use of algebra (i.e. *Financial Algebra* or *Advanced Algebra with Financial Applications*). A finance-based name is not an accurate representation of the course.]

LEVEL	GRADE	CREDIT
REGULAR	10 – 12	1.0

PREREQUISITE: Algebra 1

DESCRIPTION: Financial Algebra is an algebra-based, applications-oriented, technology dependent course that requires Algebra 1 and Geometry as a prerequisite. The course addresses college preparatory mathematics topics from Advanced Algebra, Statistics, Probability, Precalculus, and Calculus under seven financial umbrellas: Banking, Investing and Modeling a Business, Employment and Income Taxes, Automobile Ownership, Independent Living, and Retirement Planning and Household Budgeting. Students use a variety of problem solving skills and strategies in real-world contexts. The mathematics topics contained in this course are introduced, developed, and applied in an as-needed format in the financial settings covered.

Financial Algebra adheres to the following basic assumptions regarding mathematics education:

- All students will have access to calculators and computers.
- Classroom activities will be student-centered.
- All units will have increased emphasis on algebraic representations, graphical representations, and verbal representations, and the interrelationships of these three approaches.
- There is an emphasis on estimation, number sense, problem solving, and the role that reading comprehension plays in problem solving.
- Evaluation will include alternative methods of assessment.

USE OF TECHNOLOGY
Spreadsheets
Internet Research
Graphing Calculator

TEXTBOOK
Gerver, R. and Sgroi, R. *Financial Algebra*. South-Western/Cengage
Learning Mason, Ohio ©2011 ISBN -13: 978-0-538-44967-0

- **INSTRUCTOR RESOURCES [List additional reference materials that may be used throughout the year]**
  [Tip: Additional reference materials may include textbooks in the areas of Geometry, Algebra II, Precalculus, Calculus, and Statistics. This will help accurately reflect the rigor of the course.]

COURSE OUTLINE

UNIT 1 Banking (approximately 25 days)

Mathematics Topics

- Using the simple interest formula $I = PRT$ and its algebraic equivalents
- Understanding compounding via iteration
- Deriving the compound interest formula $B = (1 + \frac{r}{n})^{nt}$
- Computing compound interest with and without the formula
- Applying the compound interest formula
- Introduction to limit notation $\lim\limits_{x \to a} f(x) = b$
- Approximating e by examining the sequence $\left\{ \left(1 + \frac{1}{x}\right)^x \right\}$
- Defining the natural base e using the rational and exponential expression limit notation $\lim\limits_{x \to \infty} \left(1 + \frac{1}{x}\right)^x$
- Applying the natural base e in the continuous compounding formula $B = Pe^{rt}$
- Identifying $y = ax^b$ as exponential decay when x < 1
- Identifying $y = ax^b$ as exponential growth when x > 1
- Modeling a geometric series of the type $\sum_{b=0}^{n-1} ax^b$
- Graphing exponential functions of the type $y = ax^b$
- Analyzing rational functions and their limits of the form $\lim\limits_{x \to \infty} \frac{ax^n \pm b}{cx^m \pm d}$ where n=m, n >m, and n< m
- Using the compound interest formula to derive the present value of a single deposit investment formula $P = \frac{B}{\left(1 + \frac{r}{n}\right)^{nt}}$

- Using the compound interest formula to derive the present value of a periodic deposit investment formula $P = \dfrac{B\left(\frac{r}{n}\right)}{\left(1+\frac{r}{n}\right)^{nt} - 1}$
- Using the future value of a periodic deposit investment formula

$$B = \frac{P\left(\left(1+\frac{r}{n}\right)^{nt} - 1\right)}{\left(\frac{r}{n}\right)}$$

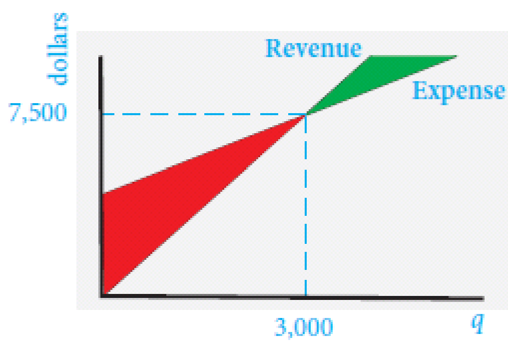
- Adapting all banking formulas for input into a spreadsheet

Essential Financial Applications
Savings accounts; compound interest; continuous compounding; future value of single and periodic investments; present value of single and periodic investments; reconcile a bank statement, annual percentage rate (APR); annual percentage yield (APY)

UNIT 2 Investing and Modeling a Business (approximately 30 days)

Mathematics Topics

- Constructing and interpreting scatterplots
- Operations with functions
- Evaluating functions and using them to model situations
- Translating verbal situations into algebraic linear functions
- Translating verbal situations into quadratic functions
- Creating rational functions of the form $f(x) = \dfrac{mx+b}{x}$
- Translating verbal situations into linear and quadratic inequalities
- Solving linear systems of equations and inequalities such as:



- Solving systems of linear equations and inequalities in two variables
- Identifying domains for which f(x) > g(x), f(x) = g(x), and f(x) < g(x)
- Identifying form, direction, and strength from a scatterplot
- Finding, interpreting, and graphing linear regression equations
- Determining domains for which prediction using a regression line is considered extrapolating or interpolating
- Finding and interpreting the Pearson Product-Moment Coefficient of Correlation
- Finding the axis of symmetry $x = \dfrac{-b}{2a}$, vertex $\left(\dfrac{-b}{2a}, f\left(\dfrac{-b}{2a}\right)\right)$, roots, and the concavity of parabolic curves
- Using the quadratic formula $if\ ax^2 + bx + c = 0\ then\ x = \dfrac{-b}{2a} \pm \dfrac{\sqrt{b^2 - 4ac}}{2a}$

- Finding and interpreting quadratic regression equations

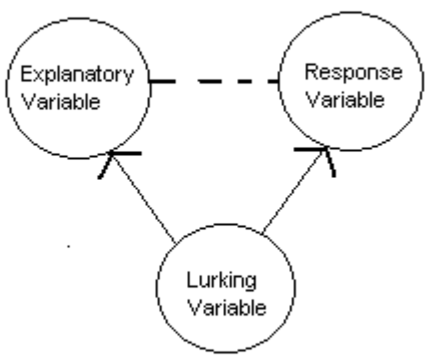
- Solving linear-quadratic systems of equations and inequalities such as:

- Finding absolute and relative extrema

- Causation vs. correlation for bivariate data

- Identifying explanatory and response variables

- Identifying and diagramming lurking variables such as:



- Using the slope-intercept form of a linear equation $y = mx + b$

- Interpreting slope as a rate of change $\frac{\Delta y}{\Delta x}$

- Using the transitive property of dependence

- Determining the zero net difference

- Writing algebraic formulas for use in spreadsheets

- Rational Expressions

- Algebraic fractions, ratios, and proportions
- Writing literal equations

- Solving linear equations and inequalities

- Calculating moving averages
- Reading and interpreting data in pictorial representations
- Algebraic representations of percent, percent increase and percent decrease
- Expressing averages as rational functions
- Translating verbal expressions into algebraic formulas for use in a spreadsheet

Essential Financial Applications

Supply and demand; fixed and variable expenses; graphs of expense and revenue functions; breakeven analysis; the profit equation; mathematically modeling a business, sole proprietorships, partnerships, candlestick charts, simple moving averages, reading and interpreting stock market ticker output, stockbroker commissions, net proceeds, gross profit, stock splits, dividend Income, yield vs. bank interest, common and preferred stock, corporate bonds

UNIT 3 Credit (approximately 25 days)

Mathematics Topics

- Using algebraic proportions
- Finding and interpreting cubic regression equations of the form $y = ax^3 + bx^2 + cx + d$
- Using slope-intercept form $y = mx + b$
- Using and interpreting exponential growth and decay equations
- Computing the average daily balance
- Applying the monthly payment formula $M = \dfrac{P\left(\frac{r}{12}\right)\left(1+\frac{r}{12}\right)^{12t}}{\left(1+\frac{r}{12}\right)^{12t} - 1}$
- Using slope-intercept form y=Mx+b where $M = \dfrac{P\left(\frac{r}{12}\right)\left(1+\frac{r}{12}\right)^{12t}}{\left(1+\frac{r}{12}\right)^{12t} - 1}$
- Using the formula $FC = \left[\dfrac{P\left(\frac{r}{12}\right)\left(1+\frac{r}{12}\right)^{12t}}{\left(1+\frac{r}{12}\right)^{12t} - 1}\right]x + b - R$ where FC = finance charge and R = retail price
- Using inverse functions to introduce the natural logarithm function $y = \ln x$ as $y = \log_e x$ and as the inverse of $y = e^x$
- Using the formula $M = \dfrac{P\left(\frac{r}{12}\right)\left(1+\frac{r}{12}\right)^{12t}}{\left(1+\frac{r}{12}\right)^{12t} - 1}$ to solve for the exponent t where

$$t = \frac{\ln\left(\frac{M}{p}\right) - \left(\ln\left(\frac{M}{p} - \frac{r}{12}\right)\right)}{12\ln\left(1 + \frac{r}{12}\right)}$$

- Modeling the average daily balance using the formula $\sum_{i=1}^{n} \frac{d_n}{n}$
- Calculating the finance charge using the formula $FC = \left(\sum_{i=1}^{n} \frac{d_n}{n}\right)\frac{APR}{12}$
- Creating algebraic formulas and applying them for use in spreadsheets

Essential Financial Applications

Credit; deferred payments; mark up, wholesale price; retail price; finance charge loans; loan calculations and regression; credit cards; credit card statement; average daily balance

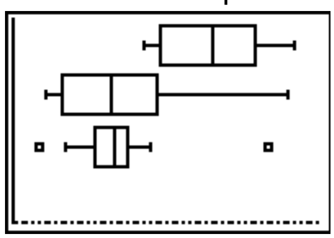
UNIT 4 Automobile Ownership (approximately 25 days)

Mathematics Topics

- Systems of linear equations
- Modeling exponential depreciation as
  $y = Px^b$ where P is purchase price and $x < 1$.
- Transforming raw data into a frequency distribution
- Creating and interpreting stem and leaf plots and side-by-side steam plots such as

9 8 1 1 1 1	87	1 2 2
3 2	88	2 4 6 7
7 5 4	89	1 3
7 6 6 6 6 5	90	2 7 7 7

- Creating and interpreting box and whisker plots and side-by-side boxplots

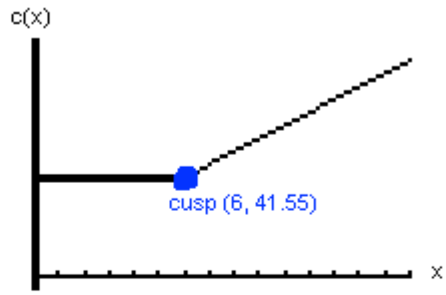


- Creating and interpreting modified box and whisker plots
- Computing measures of dispersion $R = x_H - x_L$ and $IQR = Q_3 - Q_1$.
- Computing Q_1, Q_2, Q_3, and Q_4 manually and with the graphing calculator
- Using the expressions $Q_1 - 1.5(IQR)$ and $Q_3 + 1.5(IQR)$ to determine outliers
- Compute and interpret percentiles
- Measures of central tendency $\bar{x} = \dfrac{\sum_{i=1}^{n} x_i}{n}$, median and mode
- Creating and interpreting piecewise (split) functions of the form

$$c(x) = \begin{cases} 38 \text{ when } x \le 4 \\ 38 + 6.25(x-4) \text{ when } x > 4 \end{cases}$$

- Determining the domains of a piecewise function from verbal situations
- Graphing piecewise functions using mutually exclusive domains

- Identifying the cusp of a piecewise function at a change in slope such as



- Using multi-variable square root functions such as the skid length $S = \sqrt{30Dfn}$.
- Using $RD = 0.75\left(\frac{5280s}{60^2}\right)$ to determine reaction distance
- Using $BD = 5(.1s)^2$ to compute the breaking distance
- Using $TSD = 0.75\left(\frac{5280s}{60^2}\right) + 5(0.1s)^2$ to compute total stopping distance
- Manipulating $D = RT, R = \frac{D}{T}$, and $T = \frac{D}{R}$ to determine distance, rate, and time
- Using $D = MPG(G)$ to compute miles per gallon
- Using geometry theorems involving chords intersecting in a circle and radii perpendicular to chords to determine yaw mark arc length
- Finding radius $r = \frac{C^2}{8M} + \frac{M}{2}$ where C is chord length and M is middle ordinate
- Computing arc lengths
- Using dilations D_k to transform formulas between the English Standard and Metric measurement systems
- Applying all algebraic formulas from the chapter for use in spreadsheets

Essential Financial Applications
Classified ads, negotiating auto purchases and sales, automobile insurance, linear automobile depreciation, historical and exponential depreciation, driving data, driving safety data, accident investigation data

Unit 5 Employment and Income Taxes (approximately 28 days)

Mathematics Topics

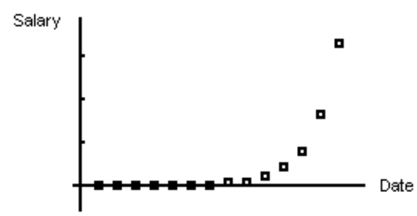
- Identifying continuous and discontinuous functions by their graphs
- Interpreting jump discontinuities
- Writing an interpreting domains and piecewise functions of the forms

$$r(x)=\begin{cases}29.95 \text{ if x is an integer and } x\le 2\\ 29.95+14(x-2) \text{ if x is an integer and } x>2\end{cases}$$

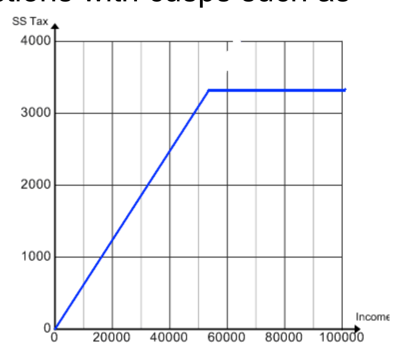
and

$$c(x)=\begin{cases}0.20x \text{ when } 0\le x<750\\ 0.22x \text{ when } 750\le x\le 1{,}000\\ 0.25x \text{ when } x>1{,}000\end{cases}$$

- Graphing exponential pay schedules such as



- Graphing piecewise functions with cusps such as

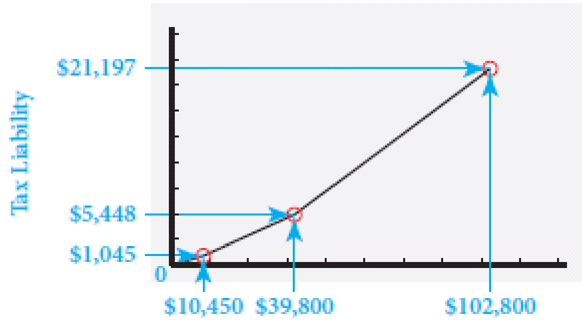


- Using measures of central tendency and rational functions such as

$$a(x)=\frac{40r+1.5tr}{t+r}$$

- Geometric sequences such as $a_n = xr^n$ with common ratio r
- Expressing percent increases and decreases as rational functions
- Reading and interpreting data
- Introducing point-slope form $y - y_1 = m(x - x_1)$ and converting it to slope-intercept form $y = mx + b$

- Graphing continuous polygonal functions with multiple slopes and cusps



- Translating verbal expressions into literal rational, exponential, and linear equations.

- Expressing domains using compound inequality notation of the form $t \geq t_1$ *and* $t < t_2$

- Expressing domains using compound inequality notation of the form $t > t_1$ *and* $t \leq t_2$, interval notation of the form $t_1 < x \leq t_2$, and tax schedule notation of the form "over t_1 but not over t_2"

- Given a compound inequality statement, modeling a tax bracket to determine the tax using a linear equation of the form $y = a + p(x - t_1)$ where y is the tax, a is the base tax, p is the tax percentage expressed as a decimal, t_1 is the lower boundary of the domain, and x is the taxable income

- Converting point-slope form to slope-intercept form of a linear equation

- Writing equations in point-slope form

- Modeling algebraically a tax schedule of the form

Schedule Y-1— If your filing status is **Married filing jointly** or **Qualifying widow(er)**

If your taxable income is: Over—	But not over—	The tax is:	of the amount over—
$0	$16,050	.... 10%	$0
16,050	65,100	$1,605.00 + 15%	16,050
65,100	131,450	8,962.50 + 25%	65,100
131,450	200,300	25,550.00 + 28%	131,450
200,300	357,700	44,828.00 + 33%	200,300
357,700	....	96,770.00 + 35%	357,700

- Using a piecewise function of the form

$$f(x) = \begin{cases} 0.10x & 0 < x \leq 16{,}050 \\ 1{,}605 + 0.15(x - 16{,}050) & 1{,}605 < x \leq 65{,}100 \\ 8{,}962.50 + 0.25(x - 65{,}100) & 65{,}100 < x \leq 131{,}450 \\ 25{,}550 + 0.28(x - 123{,}700) & 131{,}450 < x \leq 200{,}300 \\ 44{,}828 + 0.33(x - 200{,}300) & 200{,}300 < x \leq 357{,}700 \\ 96{,}770 + 0.35(x - 357{,}700) & x > 357{,}700 \end{cases}$$

where f(x) represents the tax liability function for taxpayers using a given tax schedule with taxable incomes on a given domain

- Graphing piecewise functions of the form

$$f(x) = \begin{cases} y = 0.10x & 0 < x \leq 16{,}050 \\ y = 0.15x - 802.5 & 16{,}050 < x \leq 65{,}400 \\ y = 0.25x - 7{,}312.5 & 65{,}100 < x \leq 131{,}450 \end{cases}$$

  on the coordinate plane.

- Identifying the cusps of piecewise functions from the function notation

- Interpreting the graphs, slopes, and cusps of continuous polygonal functions with multiple slopes and cusps

- Translating verbal expressions into literal equations

- Adapting all algebraic formulas in the unit for use in spreadsheets

Essential Financial Applications
Tax credits and tax deductions, tax evasion vs. tax avoidance, filing long form 1040, filing Schedules A and B, filing Forms 1040A and 1040EZ, looking for employment, pay periods and rates, commissions, royalties, piecework pay, employee benefits, Social Security and Medicare, tax tables, worksheets, and schedules, modeling Tax Schedules

UNIT 6 Independent Living (approximately 22 days)

Mathematics Topics

- Using rational functions to compute back-end ratios of the form
$$b = \frac{m + p/12 + h/3 + c + d}{a/12}.$$
- Using rational functions to compute front-end ratios of the form
$$f = \frac{m + p/12 + h/12}{x/12}.$$
- Using the monthly payment formula $M = \dfrac{\left(P\left(\frac{r}{12}\right)\left(1+\frac{r}{12}\right)^{12t}\right)}{\left(\left(1+\frac{r}{12}\right)^{12t} - 1\right)}$
- Computing interest I = $\dfrac{\left(P\left(\frac{r}{12}\right)\left(1+\frac{r}{12}\right)^{12t}\right)}{\left(\left(1+\frac{r}{12}\right)^{12t} - 1\right)} - C$ where C is original cost
- Using the apothem to compute the area of a regular polygon $A = \frac{1}{2}ap$
- Using probability to find the area of irregular plane region (The Monte Carlo Method)
$$\frac{\text{number of points inside region}}{\text{number of random points generated}} = \frac{\text{area of irregular region}}{\text{area of framing rectangle}}$$
- Using factors of dilations to draw to scale
- Finding areas of irregular and shaded regions
- Using rational functions to compute BTU's, such as $\text{BTU rating} \approx \dfrac{while}{60}$
- Solving proportions
- Creating multi-variable tax assessment equations
- Using exponential equations to model rent increases such as
$$R = A\left(1 + \frac{B}{100}\right)^{D-1}$$

- Modeling rent increases using exponential regression
- Reading and interpreting data
- Using the future value of a periodic deposit formula $B = \dfrac{P\left(\left(1+\dfrac{r}{n}\right)^{nt}-1\right)}{\left(\dfrac{r}{n}\right)}$ to make comparisons to mortgage payments and increasing resale value of a home
- Writing all algebraic formulas from the chapter for use in spreadsheets
- Translating verbal expressions into literal equations

Essential Financial Applications
Condominiums, cooperative rentals, private residences, reading a floor plan, the mortgage application process, purchasing a home, renter’s and homeowner’s insurance, liability and umbrella insurance, personal floater insurance

UNIT 7 Retirement Planning and Budgeting (approximately 25 days)

Mathematics Topics

- Using the future value of a periodic investment formula of the form

  $$B = \frac{P\left(\left(1+\frac{r}{n}\right)^{nt}-1\right)}{\frac{r}{n}}$$

  to predict balances after t years when given a periodic deposit amount, an investment return rate, and compounding information

- Using the present value of a periodic investment formula of the form

  $$P = \frac{B\left(\frac{r}{n}\right)}{\left(1+\frac{r}{n}\right)^{nt}-1}$$

  to determine the principal when given a future value, a time in years, an investment return rate, and compounding information

- Writing rational expressions as a combination of rational and polynomial expressions

- Using inequalities to define domains when creating algebraic expressions

- Analyzing the effect that a change in multipliers has to the value of an algebraic expression

- Writing rational expressions to represent increase over time

- Using and interpreting the greatest integer function of the form $[x]$

- Determining and interpreting the expected value of a probability distribution where the expected value is of the form $\sum_{i=1}^{n} x_i f(x_i)$

- Reading and interpreting data presented in multiple formats

- Creating, interpreting, and graphing greatest integer functions of the form $y = [x-a]$

- Creating, interpreting, and graphing greatest integer functions of the form $y = [x-a]+1$

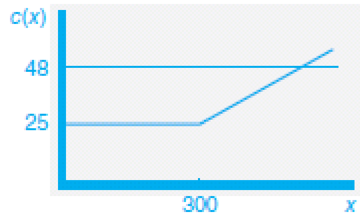
- Understanding the algebraic and contextual differences between $y = [x-a]$ and $y = [x-a]+1$

- Incorporating the greatest integer function into a piecewise function of the form

$$c(x) = \begin{cases} a & \text{when } x \le b \\ a + c(x - d) & \text{when } x > b \text{ and } x \text{ is an integer} \\ a + c([x - d] + 1) & \text{when } x > b \text{ and } x \text{ is not an integer} \end{cases}$$

- Evaluating a piecewise function that includes a greatest integer function for various values on the domain of the piecewise function

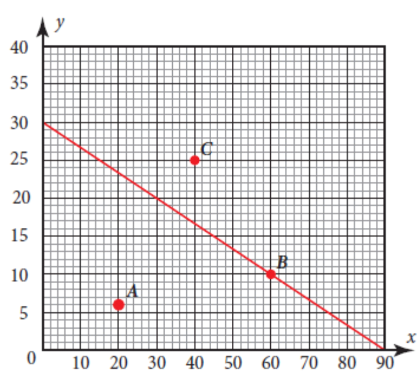
- Creating, interpreting, and graphing a system of a linear and a piecewise function and determining the point of intersection as shown in the following graph:



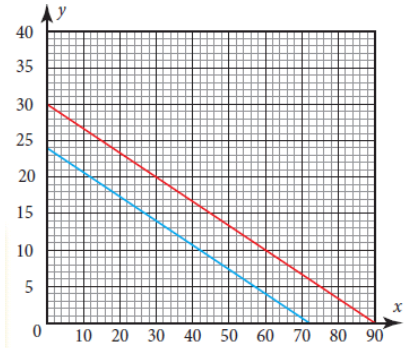
- Using sectors and central angles of a circle to depict proportional categories on a pie chart when given categorical information

- Creating and interpreting budget line equations of the type $C_x x + C_y y = B$ where C_x represents the cost of the first of two items and C_y represents the cost of the second of two items, x and y represent quantities under consideration and B represents an amount budgeted

- Interpreting points on a budget line graphs in the context of their relationship to the budget line as shown in the following display:

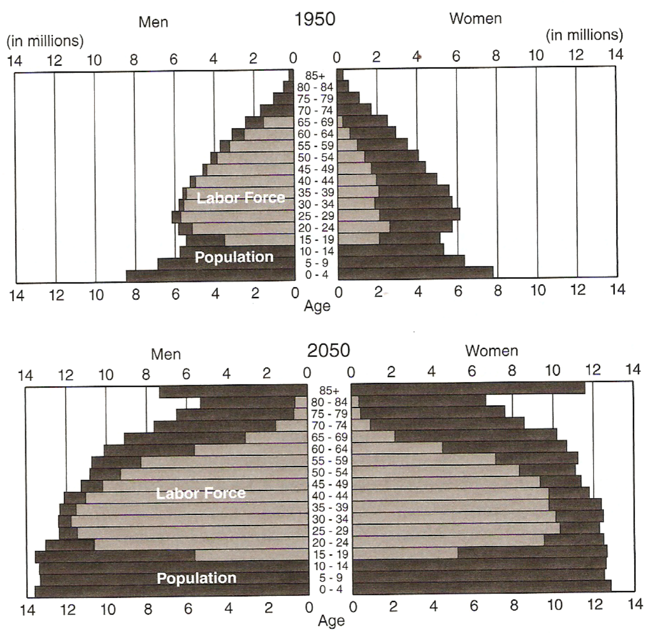


- Comparing budget line graphs and interpreting them as transformations in the plane as shown here:



- Using inequalities to interpret regions and points in the plane in relation to a budget line graph

- Using multiple representations to chart data such as



Bureau of Labor Statistics

- Using algebraic rational expressions to model ratios in context

- Writing algebraic formulas for use in spreadsheets

Essential Financial Applications

Retirement income from savings; social security benefits; pensions; life insurance; utility expenses; electronic utilities; charting a budget; cash flow; budgeting